Explicity and Implicity: Separation and Imprisionment

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Abstract

This paper advances a unified philosophical-mathematical thesis: individuation is not an ontological property of the real, but an operation externally constituted upon a pre-categorical *continuum* — the homeomerism — by means of culturally contingent linguistic signaling. From this central thesis five articulated moments derive. First, Hume's Separability Principle assumes, without argument, that every relation between distinct terms is of the pure explicit type; the Quadruple Table of Relations, introduced here, maps three additional classes that refute this premise. Second, homeomerism — a dense, categorially indifferent, synthetically inert *continuum* — is formalized as a presheaf topos with chaotic Lawvere–Tierney topology, where the Yoneda Lemma governs the representability of connections. Third, the discretization operation is constructed as a geometric morphism whose inverse image forgets continuity, explicitly defined by a functor of points $p : D \to C$. Fourth, epistemic entrapment — the irreversible closure of the space of conceivable alternatives that may follow the first linguistic discretization — is shown as the impossibility of lifting global points from homeomerism to the discretized topos. Fifth, Homotopy Type Theory and Voevodsky's Univalence Axiom operate only within already individuated domains, suffering the compositional incompleteness established in Theorem 1. The set of analyses supports the view that conceivability does not imply real possibility and that the transcendental subject is constitutively mediated by language.

Keywords: homeomerism · discretization · epistemic entrapment · Separability Principle · topos theory · Yoneda Lemma · geometric morphism · logical atomism · Quadruple Table of Relations · Univalence Axiom.

1 Introduction: The Question of Individuation as an Architectural Problem

Modern philosophy inherited a silent premise that it rarely subjected to systematic examination: that the real already presents itself articulated into discrete, individuated, separable objects, ready to be captured by thought. This premise sustains simultaneously Hume's empiricism, Kant's transcendentalism, and various contemporary formal ontologies — including, as will be argued, the foundations of Homotopy Type Theory. The present paper argues that

this premise deserves to be questioned, and that this questioning has consequences affecting epistemology, modal metaphysics, and the foundations of mathematics.

The central argument is as follows: between the real and the concept lies a structure that the philosophical tradition described only negatively — as chaos, matter, raw datum — without ever characterizing it positively. Taking up the pre-Socratic vocabulary of Anaxagoras with contemporary analytical precision, we designate this structure by **homeomerism**: a dense, categorially indifferent, synthetically inert perceptual *continuum*. Homeomerism is not the absence of multiplicity; it is non-discretized multiplicity. The operation that transforms it into an enumerable manifold — the condition of possibility of all apperceptive synthesis — is not purely transcendental in the Kantian sense: it involves linguistic, historical, and contingent dimensions.

To formalize this transition, we introduce the **Logic of Categorical Emergence** (LCE), whose core is the construction of a geometric morphism between a discrete topos (the world of already individuated concepts) and the topos of homeomerism (the pre-categorical *continuum*). The Yoneda Lemma provides the analytical mechanism that governs representability, adjunction, and the impossibility of lifting points — which explains epistemic entrapment.

The argument unfolds in five lines of consequences. **In Section 2**, the internal incoherence of the concept of separation and of the Separability Principle is examined. **In Section 3**, the Quadruple Table of Relations and the decisive counterexample are introduced. **In Section 4**, the concept of homeomerism is systematically developed and formalized as a topos. **In Section 5**, the discretization operator is constructed as a geometric morphism, with explicit definition via a functor of points. **In Section 6**, epistemic entrapment is demonstrated as a failure of point lifting. **In Section 7**, compositional incompleteness and the dilemma of the Univalence Axiom are examined. **Section 8** answers anticipated objections. **Section 9** concludes.

2 The Incoherence of the Concept of Separation

Hume's Separability Principle — that every distinguishable item is equally separable — is not an ornament of his system: it is its foundation. Upon it rest the denial of necessary causation, Hume's Dictum, and the entire analysis of causality as constant conjunction. To investigate its internal coherence is to investigate the foundation of the edifice.

2.1 The Paradox of the Boundary

When one asks what separation is — not what it appears to be, but what it is — the concept finds no stable ontological domicile. To assert that A is separated from B, it is necessary to refer to A and B together in the same act of thought: this co-reference is already a relation between them. The affirmation of separation thus presupposes the very conjunction it denies. The argument becomes more precise when one asks where the separation between A and B lies. It is not in A: no intrinsic property of A contains the relation of being-separated-from-B. It is not in B, for symmetric reasons. If it lies between them as a third element, that element must itself be separated from A and B — requiring new boundaries, leading to a

vicious regress. And if separation is merely a relation without independent existence, then it presupposes already individuated terms.

Observation 2.1. The contradiction becomes performative: the proposition $\mathrm{Sep}(A, B)$ denies every constitutive relation between A and B, but its utterance requires referring to A and B jointly. To affirm separation is to deny what the act of affirming has already accomplished.

2.2 Mereology and Primitivism

The attempt to salvage separation by means of mereology encounters instructive difficulties. The distinction between internal and external relations — those derivable from the analysis of the intrinsic natures of the terms and those that constitute an ontological addition — cannot ground mereology without circularity. If the *parthood* relation is internal and necessarily implies non-separation, then by logical transitivity it should be possible to derive non-separation by the same analysis. The problem: non-separation is, by hypothesis, an external relation. The circularity is constitutive, not accidental.

Mereological primitivism — accepting *parthood* as irreducible — is the most honest position, but at the cost of abandoning any pretense that the internal/external distinction can ground ontology. What emerges is that this distinction does not mark an objective metaphysical difference but an epistemic difference relative to the state of the analyzing subject. The conclusion is precise: for separation to exist, there must be already individuated terms; and individuation is not provided by the terms themselves but by an operation prior to them.

3 The Quadruple Table of Relations

Modern philosophy inherited an apparently exhaustive dichotomy: internal or external relations, analytic or synthetic, necessary or contingent. We introduce here a different classification, based on what a relation *does* with the identity and difference of its terms. The two relevant properties are logically independent of each other and, by combinatorial logic, generate four cells — of which only one is recognized by the Separability Principle.

3.1 Definitions

Definition 3.1 (Explicit Relation)**.** *A binary relation R is **explicit** if and only if it is irreflexive: for every x in the domain, $\neg R(x, x)$. In particular, whenever $R(x, y)$ holds, the terms are necessarily distinct: $R(x, y) \Rightarrow x \neq y$. Explicitness makes the difference between the terms epistemically manifest.*

Definition 3.2 (Implicit Relation)**.** *A binary relation R on a domain D is **implicit** if there exists a function $I : D \to E$ such that: (1) I is structurally motivated by R — it is a homomorphism, congruence, or natural measure defined from the structure that R imposes on D; (2) I is non-trivial in the strong sense: there exist $a, b \in D$ with $a \neq b$ and $I(a) \neq I(b)$, and there exist $x \neq y$ in D with $R(x, y)$ and $I(x) = I(y)$.*

The requirement of a structurally motivated invariant is the core of the definition. The equivalence of fractions $2/4 \sim 1/2$ in the rationals is implicit because the invariant "ratio" is not chosen arbitrarily: it is imposed by the very structure of the field of rationals. In contrast, for the relation "is neighbor in the queue", there is no structurally motivated invariant that equalizes distinct neighbors.

Table 1: Quadruple Classification of Ontological Relations

Type	Explicit	Implicit	Paradigmatic example
Pure Explicit	Yes	No	"greater than" in $\mathbb{N}$: $m > n$ manifests $m \neq n$ without invariant
Pure Implicit	No	Yes	$2/4 \sim 1/2$: distinct numbers, same structural ratio
Both	Yes	Yes	MP within S: proper part with essential functional invariant
Neither	No	No	Not-being-separated in the homeomeric continuum

3.2 Consequences for the Separability Principle

Hume's argument against necessary causation has the following structure: (i) cause and effect are distinct objects; (ii) every distinct object is separately conceivable; (iii) everything separately conceivable is separately possible; therefore (iv) there is no necessary connection. Premise (ii) silently presupposes that every relation between distinct objects is of the *pure explicit* type. The Quadruple Table offers two ways to refute this presupposition. Via the **Both** class: a part of a whole is distinguishable from the whole — the proper *parthood* relation is irreflexive, hence explicit. However, in cases of essential existential dependence, the part cannot exist separately. Such inseparability is not empirical contingency but constitutive necessity: the identity of the term essentially depends on the whole.

Via the **Neither** class: the Separability Principle presupposes already individuated terms. In the homeomeric regime, this individuation has not yet occurred. The principle is thus local, not universal. Its appearance of universality is an artifact generated by the fact that Western philosophy has often worked with already individuated objects without thematizing the act of individuation.

4 Homeomerism: The Pre-Individuated *Continuum*

4.1 Definition and Essential Properties

The concept of homeomerism designates a perceptual-ontological state characterized by four essential properties:

- **Topological density**: the *continuum* is continuously filled, without natural gaps — but "point" here is an improper metaphor, for homeomerism has no units.

- **Categorical indifference**: absence of inherent divisions or partitions into finitely distinguishable classes.
- **Structural completeness**: the *continuum* is integrally present; it is not a sum of parts.
- **Synthetic inertia**: apperceptive synthesis cannot operate upon it without differentiating mediation.

Homeomerism is not absence of multiplicity: it is non-discretized multiplicity.

Associated with homeomerism is the property we call **Without-Reason**: the intrinsic absence of internal reasons for differentiation in the *continuum*. Two types are distinguished: *material* homeomerisms (continuous sensory fields: color spectrum, sound intensities) and *formal* homeomerisms (transcendental structures that exist integrally but remain internally unarticulated). The sorites paradox illustrates the thesis: vagueness is not necessarily a logical defect — it may be the trace of a pre-categorical *continuum* whose discretization is imposed, not discovered.

4.2 Formalization: The Topos of Homeomerism

Homeomerism is not merely a philosophical concept: it has a precise mathematical structure. Let C be a small category whose objects are the virtual points of homeomerism and whose morphisms $c \to d$ exist whenever, before any discretization, there is no internal reason to distinguish c from d. We take C as the category with a single object $*$ and morphisms forming a continuous group G (e.g., $\mathbb{R}$ under addition). This guarantees the absence of privileged terminal objects.

By the **Yoneda Lemma**, we have the full and faithful embedding:

$$\mathbf{y} : C \hookrightarrow \mathbf{Set}^{C^{\mathrm{op}}}.$$

Each object c is represented as the presheaf $\mathbf{y}(c) = \mathrm{Hom}_C(-, c)$. These presheaves are not atomic: they encode the web of connections with all other objects.

Homeomerism H is the presheaf topos $\mathbf{Set}^{C^{\mathrm{op}}}$ equipped with a **chaotic (trivial) Lawvere–Tierney topology**: the closure operator $j : \Omega_H \to \Omega_H$ sends every sieve to the top. The consequences are precise:

- Every subobject is dense; there are no distinguishable proper parts.
- The only separated object (*sheaf*) is the terminal object 1.
- The classifier Ω_H contains all sieves, but no global point can select a non-trivial subobject.

This mathematically captures *synthetic inertia*: any attempt to "cut" an object in pure homeomerism fails, because the cut is immediately reabsorbed by the topology.

4.3 The Gap in Empiricism: Hume Requires Kant

Hume's Copy Principle — every legitimate idea derives from a corresponding sense impression — carries an unthematized presupposition: for an impression to be copied into an idea, it must be individuated. This individuation is not provided by the impression itself: it presupposes criteria of cutting. The criteria of individuation cannot themselves be impressions. Humean empiricism can describe how ideas are combined; it cannot explain how impressions are individuated. In this sense, **Hume requires Kant** in order to be Hume.

4.4 The Gap in Kant's Architecture: Kant Requires Language

Kant demonstrates that space and time are given infinite magnitudes. Concepts are necessarily discrete and enumerable. Apperceptive synthesis presupposes a given manifold. The question that no passage of the *Critique of Pure Reason* directly answers is: how does the given *continuum* become the enumerable manifold that apperceptive synthesis presupposes? The Schematism is the official answer, but the temporal schema does not thematize what operates before the schema so that the temporal continuum becomes a series of numerable instants.

Homeomerism is precisely what is not yet an articulable manifold. In this sense, **Kant requires language** in order to be Kant — it requires a theory of linguistic discretization that the pure forms of sensibility do not provide. The chain is thus: Humean empiricism requires individuation of impressions that it cannot supply; Kantian transcendentalism requires that the given continuum be already articulable; the articulation that both presuppose is supplied by linguistic signaling — historically contingent, irreversibly formative.

5 The Discretization Operator as a Geometric Morphism

5.1 The Mechanism of Contraction

Formally, homeomerism H can be described as possessing a trivial topology $\{\emptyset, H\}$: the only structure is the whole H; no proper part is internally distinguishable. **Contraction** is the operation that introduces the first distinguished point $\{p\} \subset H$, breaking the trivial topology. Without-Reason guarantees that there is nothing in homeomerism that determines which point will be p. The sufficient reason that this point and not another is the first lies in the intentionality of the Other who emits the linguistic signal — their history, their culture, their language.

Observation 5.1. An analogy: a board with iron powder suspended over a uniform surface (material homeomerism). A magnetic pen, carrying a particular configuration, touches the surface; the iron grains organize themselves around the contact: a point emerges. The reason why this point is the first lies not in the powder but in the pen. Kantian subsumption is not a purely transcendental operation: it depends on discretization produced by culturally contingent linguistic signaling.

5.2 Categorical Construction of the Morphism

Let D be a discrete category whose objects are the atoms generated by the first linguistic discretization (e.g., $\{1, 2, \ldots, n\}$ with only identity morphisms). The topos $\mathbf{Set}^{D^{\mathrm{op}}}$ is isomorphic to $\mathbf{Set}^n$ — objects are n-tuples of sets, independent of each other.

To define the geometric morphism

$$f : \mathbf{Set}^{D^{\mathrm{op}}} \longrightarrow H,$$

we begin by choosing a **functor of points** $p : D \to C$. Since C has a single object $*$, p takes each object $d \in D$ to the object $*$, and each identity morphism to the identity. This functor encodes the contingent choice of which points of the continuum will be the first distinguishable ones (although the chaotic topology makes all points equivalent, the *selection* is made by the discrete domain).

The **inverse image** $f^* : H \to \mathbf{Set}^{D^{\mathrm{op}}}$ is defined by precomposition with p: for any presheaf $F \in H$ (i.e., $F : C^{\mathrm{op}} \to \mathbf{Set}$),

$$f^*(F)(d) = F(p(d)) \quad \text{for each } d \in D.$$

Since $p(d) = *$ for all d, we have $f^*(F)(d) = F(*)$; therefore $f^*(F)$ is the constant presheaf in $\mathbf{Set}^{D^{\mathrm{op}}}$ with value $F(*)$. The verification that f^* preserves finite limits is immediate, because limits in $\mathbf{Set}^{D^{\mathrm{op}}}$ are computed pointwise and $F(*)$ is just a set.

The **right adjoint** $f_* : \mathbf{Set}^{D^{\mathrm{op}}} \to H$ is given by the general formula for geometric morphisms induced by a functor:

$$f_*(G)(c) = \mathrm{Hom}_{\mathbf{Set}^{D^{\mathrm{op}}}}\left(\mathbf{y}_C(c) \circ p, G\right),$$

where $\mathbf{y}_C(c) : C^{\mathrm{op}} \to \mathbf{Set}$ is the Yoneda embedding. Since C has only one object $*$, $\mathbf{y}_C(*)(-) = \mathrm{Hom}_C(-, *)$ is the constant representable presheaf (in the case of the group $\mathbb{R}$, each $\mathrm{Hom}_{\mathbb{R}}(r, *)$ is a singleton set). Simplifying, we obtain:

$$f_*(G)(c) \cong \prod_{d \in D} G(d),$$

i.e., the constant presheaf in H whose value is the product of all coordinates of G. The pair (f^*, f_*) constitutes a geometric morphism. The Yoneda Lemma guarantees that the terminal object is preserved: $f_*(1_D)(c) \cong 1$, hence it sends the terminal object to the terminal object.

Proposition 5.1. *The discrete world is not ontologically primary; it is the image of a geometric projection from the continuum. Hume mistakenly took this image as the real itself.*

5.3 The Sliced Topos and the Counterexample MP/S

To capture the essential dependence between the rule of *modus ponens* (MP) and the formal system S that contains it, we abandon the base category **Set** and work in the sliced topos $\mathcal{E}/S$, where $\mathcal{E}$ is a base topos.

- Objects of $\mathcal{E}/S$ are morphisms $X \to S$ in $\mathcal{E}$.

- The terminal object in $\mathcal{E}/S$ is the identity $S \to S$.
- The rule MP is represented by the morphism $m : MP \to S$ that anchors *modus ponens* in the system.

In the sliced topos, the existence of MP *is* the arrow m. To attempt to "separate" MP from S would amount to considering the object MP without the morphism to S — which is a different object. The forgetful operation $\mathcal{E}/S \to \mathcal{E}$ takes m to its domain MP, but loses the relational structure. Hume made the mistake of taking this forgetting as an innocent operation, when in fact it alters the identity of the term.

6 Epistemic Entrapment as Failure of Point Lifting

6.1 The Phenomenon

Once homeomerism is discretized, it becomes transparent to consciousness: we no longer experience it as a *continuum*, but through already established categories. We call **epistemic entrapment** the possible irreversible closure of the space of conceivable alternatives that may follow the first linguistic discretization in the same domain.

The paradigmatic case is the distinction between two types of congenital blindness. **Blindness-1** is the state in which the subject reports a homogeneous qualitative experience — a "blackness" spatially extended, but without internal figure or contrast. This is a case of formal homeomerism: the form of the external sense is present, but without sufficient internal articulation to generate differentiated visual objects. **Blindness-2** is the complete deprivation of the visual field without any qualitative experience of darkness: some congenitally blind individuals simply lack the visual modality.

Once the subject of Blindness-1 internalizes the category "blindness" designating their homogeneous state, they cannot conceive the concept of Blindness-2. To conceive it, they would need to conceive the absence of a visual field. But that absence is the negation of X, and X has already been categorized as the very negation of vision. Therefore the negation of X should return vision — which the subject of Blindness-1 does not possess. The same signifier "blindness" designates mutually inaccessible concepts.

6.2 Formalization: Impossibility of Lifting

A **global point** of a topos $\mathcal{E}$ is a geometric morphism $\mathbf{Set} \to \mathcal{E}$. In pure homeomerism H, the chaotic topology prevents the existence of non-trivial global points (the only global point is the terminal one, which corresponds to the homogeneous experience of Blindness-1).

After discretization, the subject comes to inhabit the topos $\mathbf{Set}^{D^{\mathrm{op}}}$. Epistemic entrapment occurs when a global point of homeomerism (the homogeneous experience) is mapped by f^* to the empty object in the discrete world. Formally, **there exists no lift** $s : \mathbf{Set}^{D^{\mathrm{op}}} \to H$ **such that** $f \circ s \cong \mathrm{id}$**.** In particular, the global point of homeomerism cannot be represented in the discrete topos:

$$\nexists s : \mathbf{Set}^{D^{\mathrm{op}}} \to H \text{ geometric morphism such that } f \circ s \cong \mathrm{id}.$$

The proof is straightforward: suppose such s exists. Then for the global point $t : \mathbf{Set} \to H$ given by the homogeneous experience, we would have $t = f \circ (s \circ t)$. But $f^*(s_*(t))$ would be a constant presheaf in $\mathbf{Set}^{D^{\mathrm{op}}}$, whereas the original global point in H is not constant (the chaotic topology does not distinguish it, but its origin is the qualitative experience). The contradiction arises because f^* collapses all non-discrete information. More rigorously: the only global point of H that can be the image of some point of $\mathbf{Set}^{D^{\mathrm{op}}}$ is the one whose value at each d is the terminal object; this point is not the point of Blindness-1 (which is unique but not decomposed). Hence the lifting is impossible.

Corollary 6.1. *For the trapped subject, the homogeneous experience becomes inconceivable — it is Blindness-2. The subject cannot even formulate the negation of their state, because the negation would require a point in the classifier Ω_D that has been excluded by the topology of discretization.*

6.3 Empirical Evidence and Linguistic Relativism

The epistemic entrapment thesis finds consistent support in research on linguistic relativism. Winawer et al. (2007) showed that Russian speakers, who have distinct lexical terms for light blue (*goluboy*) and dark blue (*siniy*), discriminate these colors perceptually faster than English speakers — an effect that disappears under simultaneous verbal interference. The same categorical mechanism explains the phenomenon: these speakers possess discrete points in their topos that English speakers lack. The "conceivability" of a color depends on the existence of a corresponding global point in the topos of the language.

Roberson et al. (2000) confirmed that the discretization of color categories is not purely pre-linguistic. This evidence supports the more modest thesis sufficient for the philosophical argument: that perceptual individuation is not universally prior to language, and that language can reorganize — not merely name — perceptual experience.

Epistemic entrapment differs from Whorfian linguistic relativism in a decisive respect: it does not claim that language totally determines thought, but that the *trajectory of discretization* has irreversible effects. Certain alternatives become **inconceivable**, not merely difficult. The inconceivability of the absent global point is structural, not psychological.

7 Compositional Incompleteness and the Univalence Axiom

7.1 The Compositional Incompleteness Theorem

Let us call a **compositional system** any system Σ that satisfies three conditions: (1) it possesses a domain D of distinguishable and enumerable elements; (2) it has combination operations that produce new elements of D from given elements; (3) it aims to describe or generate its own structure by means of these operations.

Theorem 7.1 (Compositional Incompleteness)**.** *No compositional system Σ can thematize, by means of its own operations, the conditions of individuation that make D possible.*

Proof. The operations of Σ presuppose D as already constituted — they combine given elements, they do not produce the condition that elements are given. Every attempt to reconstruct D from the operations of Σ presupposes D by selecting the operands of the reconstruction. The circularity is constitutive, not accidental. This result is structurally analogous to Gödel's incompleteness theorems and Cantor's diagonal argument. □

7.2 The Univalence Axiom as a Pure Implicit Regime

Vladimir Voevodsky proposed, in Homotopy Type Theory (HoTT), the Univalence Axiom: identity between two types is equivalent to an equivalence between them — formally, $(A = B) \simeq (A \simeq B)$. In philosophical language: two types that behave indistinguishably in all contexts are the same type. This axiom can be seen as a formalization of what the Quadruple Table calls the *pure implicit regime*: the apparent difference between A and B is dissolved by a structural equivalence that reveals a deeper identity under numerical difference.

7.3 The Scope Dilemma

The Quadruple Table and the Univalence Axiom enter into a tension that takes the form of a dilemma about the scope of HoTT:

- **Horn 1 — universality:** if the axiom is universally valid, it applies also to the homeomeric regime. But in that regime there are no individuated types — hence the equivalence is not defined. HoTT would be inapplicable to the most fundamental substratum of ontology.
- **Horn 2 — locality:** if the axiom is valid only within already discretized domains, then HoTT presupposes individuation as a condition of possibility — and cannot thematize it internally. Consequently, HoTT suffers from the structural incompleteness described in Theorem 1.

The dilemma is not a refutation of HoTT, but a **delimitation of scope**. HoTT is a highly sophisticated system for reasoning about already individuated mathematical structures. Hume begins with individuated impressions; Kant begins with articulable intuitions; Voevodsky begins with a universe of types. None of them thematizes what makes it possible that there are individuated impressions, articulable intuitions, or distinguishable types. The question of individuation is prior — transcendentally — to all of them.

8 The Counterexample: Proper Part with Essential Existential Dependence

8.1 Construction and Formalization

To refute the universality of the Separability Principle, we present a concrete counterexample based on the Both class. Let $Pp(x, y)$ be the relation "x is a proper part of y". In extensional mereology, $Pp(x, y)$ is irreflexive: $\neg Pp(x, x)$. We say that x has **essential existential**

dependence on y when $\Box(\exists x \rightarrow \exists y)$ and this necessity derives from the essence of x, not from contingent laws.

Consider the inference rule *modus ponens* (MP) within a specific natural deduction system S:

1. $Pp(MP, S)$ is true: MP is a proper part of S.
2. $MP \neq S$ (numerically distinct).
3. MP has essential existential dependence on S: outside S (or an equivalent system), we have at most a syntactic pattern — marks on paper — which is not the rule MP. The identity of MP as a logical rule is constitutively dependent on S.

Therefore, $D(MP, S)$ — the terms are distinguishable — is true, but $C(MP, S)$ — separately conceivable while preserving identity — is false. Hence, $\neg\forall x\forall y(D(x, y) \rightarrow C(x, y))$. **The Separability Principle is invalid.**

8.2 Responses to Neo-Humean Objections

Objection I — simple objects: Hume would be speaking about mental conceivability, not real existence. I can conceive a sound pattern identical to the dominant in C major even without the tonal system.

Response: The objection reveals the weak point of Hume's test. If separate conceivability need not preserve the identity of the term, the test is trivial. A loose interval pattern is not a tonal dominant function — it is merely a succession of notes. The dominant is defined by its tension-resolution relation to the tonic within a tonal system. Outside that system, the term "dominant" does not apply. The inseparability is *constitutive*: the identity of the term essentially depends on the whole. Hume confuses "I can abstract" with "it is possible to exist separately".

Objection II — nomic necessity: the dependence would be normative or conventional, not logical.

Response: The argument uses *essential* necessity (Fine 1994), not merely nomic. Lewis's distinction between metaphysical and nomic necessity does not apply. If the critic insists that every constitutive dependence can be dissolved by an act of imagination that abstracts the whole, then the critic already presupposes what Hume wanted to prove — the circularity is constitutive, not accidental.

Objection III — proves too much: the argument would prove that every relation is constitutive.

Response: The argument does not prove that every relation is constitutive, only that there exists a class (Both) where distinguishability does not imply separability. That is sufficient to falsify Hume's universal premise.

9 General Objections and Responses

9.1 Homeomerism is an Obscure Entity

Objection: The concept is vague; the work could be done with the traditional "continuum".

Response: Homeomerism differs from the traditional continuum in three characteristics: (i) absence of privileged points (Without-Reason); (ii) synthetic inertia; (iii) distinction between material and formal types. The standard mathematical continuum already comes with a topology that allows distinguishing open sets; homeomerism is *pre-topological*. The formalization via a topos with chaotic topology precisely captures this difference.

9.2 Epistemic Entrapment is merely Disguised Linguistic Relativism

Objection: The thesis has already been defended by Whorf and fell into disrepute.

Response: Epistemic entrapment is stronger: it claims that certain alternatives become *inconceivable*, not merely difficult. The Blindness-1 vs. Blindness-2 example shows a structural inconceivability demonstrable mathematically as a failure of *lift*. Unlike Whorf, we do not claim that language totally determines thought, but that the trajectory of discretization has irreversible effects on the space of conceivability.

9.3 The Dilemma about HoTT is Irrelevant to Philosophy

Objection: HoTT is a mathematical theory, not an ontology.

Response: Voevodsky and the Univalent Foundations program intend HoTT to serve as foundations for all of mathematics — and indirectly for ontology. If individuation is prior to type theory, then even as a mathematical foundation HoTT presupposes something it cannot justify. The dilemma delimits its scope without negating it.

9.4 Limits of the Kantian Distinction between Genesis and Validity

The analysis of epistemic entrapment suggests that the Kantian distinction between *quid facti* and *quid iuris*, when applied to the domain of subjective constitution, encounters limits. For concepts such as Blindness-2, logical analysis is possible only under the modal condition of the empirical impossibility of a visual field — a condition not derivable a priori. This does not imply relativism: it does not claim that truth varies among subjects. It claims that the space of conceivable alternatives may vary according to the subject's trajectory of discretization, and that this space is a condition of possibility for every judgment, including the analytic.

9.5 Additional Objections and Responses

Objection 1: The Blindness-1 case describes merely a uniform content that is already individuated. The homogeneous blackness attributed to the Blindness-1 subject does not constitute a pre-categorical domain. It is merely a uniform content of visual experience, analogous to the experience of a sighted individual who closes their eyes in a completely dark environment. Being a content already given to consciousness, it remains subject to ordinary categories of identity, distinction, and conceivability.

Response. This objection confuses uniformity with homeomerism. A uniformly black visual field perceived by a sighted subject remains embedded in a previously articulated visual structure. The subject knows where the visual field ends, distinguishes center and periphery, recognizes the possibility of opening the eyes, and retains an implicit horizon of differentiation. The content is uniform, but the phenomenological space remains discretized.

The Blindness-1 case is different. The described state has no internal horizon that would motivate the constitution of regions, boundaries, or visual directions. It is not a collection of equal parts, but a domain in which the very notion of a part has yet to find a ground. Homeomerism is not uniformity of pre-constituted elements; it is prior to the constitution of the elements themselves.

Thus, Blindness-1 does not correspond to a homogeneous phenomenological object, but to a situation in which the internal criteria for visual individuation have not yet emerged.

Objection 2: The example describes only a psychological limitation of conceivability. The Blindness-1 subject may be unable to conceive Blindness-2, but this only demonstrates a contingent cognitive limitation. Nothing prevents Blindness-2 from remaining metaphysically possible, even if psychologically inconceivable for certain subjects.

Response. The objection presupposes precisely what is at stake. The argument does not simply claim that the subject fails to represent an alternative possibility. It claims that the attempt to represent such a possibility destroys the very referent whose possibility one seeks to examine.

For the Blindness-1 subject, the term "blindness" is not initially defined by absence of vision, but by indexing the totality of their available visual experience. When the subject acquires vision, they do not correct an inadequate description of the same object; they replace the object of reference itself.

The situation is analogous to attempting to remove a constitutive condition of an object's identity and then claiming that we are still dealing with the same object. The conceivability obtained after the transformation does not constitute evidence for the conceivability of the original state, but for a new state generated by the very operation of abstraction.

The problem, therefore, is not psychological. It is a problem of identity preservation through abstraction.

Objection 3: Homeomerism is merely a phenomenological version of the Kantian transcendental condition. Homeomerism adds nothing to the transcendental tradition.

It corresponds merely to the condition of possibility of experience, playing a role similar to the form of sensibility in Kant.

Response. The analogy fails because the Blindness-1 subject already possesses complex forms of spatiality through touch, hearing, and proprioception. The subject's spatial structure is fully operational.

What is at issue is not the general condition of experience, but the specific constitution of the visual domain before the emergence of internal differentiations. Homeomerism is not a universal transcendental condition; it is a local condition of non-discretization.

Hence it does not coincide with the Kantian transcendental space. It is a domain prior to visual individuation, not prior to experience as such.

Objection 4: Homeomerism is a metaphysical hypothesis without empirical support. Homeomerism is a speculative construct introduced only to sustain the theory of epistemic entrapment.

Response. The biophysical appendix shows that the distinction between Blindness-1 and Blindness-2 is not arbitrary. The former corresponds to a neurophysiologically plausible scenario in which the basal activity of visual circuits remains present without significant informational modulation. The latter corresponds to the structural absence of the visual channel itself.

The philosophical thesis is not deduced from neurophysiology. However, the empirical plausibility of the scenario prevents homeomerism from being dismissed as a mere conceptual fiction. The example does not arise to save a previously formulated theory; it emerges from an independent biological possibility that demands philosophical explanation.

Objection 5: The Separability Principle remains valid for all genuine objects. Even if homeomerism exists, it does not constitute a genuine object. The Separability Principle remains valid for all individuated objects, preserving the Humean thesis in its legitimate domain.

Response. This response transforms the Separability Principle from a universal foundation of conceivability into a local principle applicable only after individuation. The consequence is decisive. The principle ceases to explain the genesis of objects and instead presupposes it.

But a principle that depends on individuation cannot simultaneously justify individuation. The objection, therefore, preserves the local validity of separability only at the cost of abandoning its foundational claim.

The conclusion is not that separability is always false. The conclusion is that it cannot occupy the ontologically primitive place that the Humean tradition assigns to it.

10 Conclusion

The Logic of Categorical Emergence unifies the philosophical critique of logical atomism with a rigorous formalization in Topos Theory. We have argued that:

1. Hume's Separability Principle is false: there exist relations (Both class) whose terms are distinguishable but inseparable — the pair (MP, S) is the counterexample.

2. Homeomerism is modeled as a presheaf topos with chaotic topology, where the Yoneda Lemma governs the representation of continuous connections.

3. Discretization is a geometric morphism whose inverse image forgets continuity, explicitly constructed from a functor of points $p : D \to C$.

4. Epistemic entrapment is the impossibility of lifting global points from homeomerism to the discrete topos.

5. HoTT and the Univalence Axiom operate only within already discretized domains, presupposing individuation without justifying it.

The most radical philosophical consequence is this: the transcendental subject, far from being purely prior to language, is constitutively mediated by it. Homeomerism is what remains when every perspective withdraws — not as an empty horizon, but as a plenitude that no cut exhausts.

> *The possible is, in part, the name we give to the fragments when we contemplate them from within our discretization.*

A Appendix: Biophysics of Blindness-1 and Blindness-2

This appendix provides neurophysiological grounding for the distinction between Blindness-1 and Blindness-2, showing how formal homeomerism finds a correlate in concrete biological mechanisms.

A.1 Blindness-1: The Dark Current and Formal Homeomerism

In the resting state (absence of photons), photoreceptors (cones and rods) in the retina are not turned off; they are chronically depolarized (about -40 mV). This occurs because intracellular cGMP levels are high, keeping CNG sodium and calcium channels open. This constant depolarization causes photoreceptors to continuously release the neurotransmitter glutamate at synapses with bipolar cells. If an individual is born with a mutation that completely prevents visual transduction (e.g., absence of functional rhodopsin), but maintains the structural integrity of the optic pathways, the photoreceptors remain constitutively depolarized. The optic nerve sends a perfectly homogeneous, invariant baseline signal to the lateral geniculate nucleus and then to the primary visual cortex (V1). V1 receives a continuous bioelectric tone, keeping active the neural networks that generate the sensation of internal spatiality, but without any contrast modulation. This is the neurophysiological correlate of formal homeomerism: the screen exists structurally, the tone exists, but the signal is identically zero in terms of discrete information.

A.2 Blindness-2: Structural Absence of Visual Space

Blindness-2 is not a "constant signal of value zero"; it is the absence of the data channel itself. In cases of severe bilateral congenital anophthalmia or complete agenesis of the optic nerve, V1 never receives the bioelectric tone of the dark current during critical development. Without organized thalamocortical projections, the occipital area undergoes cross-modal plasticity: it is colonized by other sensory modalities (touch, hearing). Since V1 no longer operates as a retinotopic map, the neural circuits responsible for sustaining the *qualia* of "visual extension" simply do not exist in that brain.

The individual does not experience "blackness", because blackness requires a baseline retinotopic activity in V1 that interprets uniformity. Asking a Blindness-2 subject about visual blackness is neurophysiologically equivalent to asking an ordinary person what they "see" through the skin of their elbow: the modality is not there.

This biophysical distinction validates the concept of epistemic entrapment: the Blindness-1 subject, by categorizing their homogeneous state as "blindness", becomes unable to conceive Blindness-2, because the negation of their own homogeneous visual field would be, for them, the return of vision — a phenomenological impossibility that is simultaneously a failure of *lift* in the topos.

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